International Journal of Theoretical Physics, Vol. 42, No. 5, May 2003 (© 2003)

Destruction in Bipartite Quantum Systems

Paweł Caban,[1,2] Jakub Rembieliński,[1] Kordian A. Smoliński,[1] and Zbigniew Walczak[1]

Received April 9, 2003

The description of destruction in bipartite quantum systems in terms of quantum mechanics rather than quantum field theory is presented. The maps called supertraces are defined and used in the definition of the destruction procedure, which can be treated as a supplement to the von Neumann–Lüders reduction postulate. The presented formalism is illustrated by several examples that may be helpful in a description of Einstein–Podolsky–Rosen type experiments and in quantum information theory.

KEY WORDS: Einstein–Podolsky–Rosen *Gedankenexperiment*; quantum information; state destruction.

1. INTRODUCTION

In this paper we describe examples of destruction of bipartite systems on the level of quantum theory with finite degrees of freedom on a basis of the destruction procedure introduced by Caban *et al*. (2002). Questions involving destructions of this kind of systems arise when Einstein–Podolsky–Rosen type experiments (Bohm, 1951; Einstein *et al*., 1935) are studied. In this type of experiments two particles are produced in an entangled state and sent to two measurement devices in the distance where correlated quantities are measured at the same time. Prediction of the correlation between the data does not cause any problems in such an ideal experiment, but if both measurements are not really performed at the same time we have to take into account that a particle is irreversibly absorbed by a detector during the measurement. This has nothing in common with an annihilation of a particle in quantum field theory; therefore, to avoid any confusion we shall use the word "destruction" to name this kind of processes.

Evidently, if we take into account the destruction we have to consider open quantum systems. We make the idealization relying on the assumption that the

[1] Department of Theoretical Physics, University of Łódź, Łódź, Poland.
[2] To whom correspondence should be addressed at Department of Theoretical Physics, University of Łódź, ul. Pomorska 149/153, 90-236 Łódź, Poland; e-mail: p.caban@merlin.fic.uni.lodz.pl.

destruction process is instantaneous, therefore, its description should not involve any dynamics.

Destruction of a particle in a detector usually occurs when some quantum numbers (e.g. spin, position, or momentum) of the particle belong to a specified subset of spectrum of the corresponding observable. Therefore, we must have a quantum system and a detector that checks if the particle quantum numbers are inside a given subset of spectrum. If the answer is "yes," the particle is destroyed.

The paper is organized as follows. In Section 2 we discuss the space of states necessary for the description of destruction in bipartite quantum systems. In Section 3 we define supertraces that are our basic tool in the definition of the destruction procedure. The Sections 4 and 5 deal with the destruction of two-particle systems of distinguishable and identical particles, respectively. We illustrate each of these cases by examples.

2. THE SPACE OF STATES

First, we discuss the space of states necessary for the description of destruction of two-particle states of particles "a" and "b." Let $\mathcal{H}_a$ and $\mathcal{H}_b$ be the Hilbert spaces for the particle "a" and "b," respectively. The two-particle Hilbert space is the tensor product $\mathcal{H}_a \otimes \mathcal{H}_b$. The state of the system is then described by the density matrix $\rho \in \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b)$. If one introduces in $\mathcal{H}_a$ and $\mathcal{H}_b$ orthonormal bases $\{|a\rangle\}$ and $\{|b\rangle\}$, respectively, then the density matrix ρ can be written in the form

$$\rho = \sum_{aa'bb'} \rho_{aba'b'}(|a\rangle \otimes |b\rangle)(\langle a'| \otimes \langle b'|) = \sum_{aa'bb'} \rho_{aba'b'}|a\rangle\langle a'| \otimes |b\rangle\langle b'|. \qquad (1)$$

In the case of identical particles the two-particle Hilbert space is the symmetric or antisymmetric part of $\mathcal{H}_a \otimes \mathcal{H}_b$, thus the coefficients $\rho_{aba'b'}$ must fulfill the following symmetry conditions

$$\rho_{aba'b'} = \rho_{baa'b'} = \rho_{abb'a'} = \rho_{bab'a'}, \qquad (2a)$$

$$\rho_{aba'b'} = -\rho_{baa'b'} = -\rho_{abb'a'} = \rho_{bab'a'}, \qquad (2b)$$

for symmetric and antisymmetric case, respectively.

But such a description of composite quantum system is not enough if we consider the measurement by the apparatus that can destroy the state. The reason is that the density matrix (1) can describe only the two-particle states of the system, while after such a measurement we could have also a one-particle state and a vacuum state.

According to Caban *et al.* (2002) we solve this issue introducing the vacuum vector $|\mathrm{vac}\rangle$ orthogonal to any vector from $\mathcal{H}_a$ or $\mathcal{H}_b$ and the one-dimensional vacuum space $\mathcal{H}^0 \equiv \{c|\mathrm{vac}\rangle\colon c \in \mathbb{C}\}$, and taking the direct sums $\mathcal{H}_a \oplus \mathcal{H}^0$ and $\mathcal{H}_b \oplus \mathcal{H}^0$ instead of $\mathcal{H}_a$ and $\mathcal{H}_b$, respectively. The corresponding tensor product space can be decomposed in the obvious way

$$(\mathcal{H}_a \oplus \mathcal{H}^0) \otimes (\mathcal{H}_b \oplus \mathcal{H}^0) \\ = (\mathcal{H}_a \otimes \mathcal{H}_b) \oplus ((\mathcal{H}_a \otimes \mathcal{H}^0) \oplus (\mathcal{H}^0 \otimes \mathcal{H}_b)) \oplus (\mathcal{H}^0 \otimes \mathcal{H}^0), \tag{3}$$

where $\mathcal{H}_a \otimes \mathcal{H}_b$ describes two-particle states, $(\mathcal{H}_a \otimes \mathcal{H}^0) \oplus (\mathcal{H}^0 \otimes \mathcal{H}_b)$ represents one-particle states, while $\mathcal{H}^0 \otimes \mathcal{H}_0$ is the zero-particle state. In the case of distinguishable particles we can take the terms $\mathcal{H}_a \otimes \mathcal{H}^0$ or $\mathcal{H}^0 \otimes \mathcal{H}_b$ as the Hilbert space of the system after destruction of the particle "b" or "a," respectively. For identical particles we have to consider the one-particle Hilbert space as a subspace of the sum $(\mathcal{H} \otimes \mathcal{H}^0) \oplus (\mathcal{H}^0 \otimes \mathcal{H})$, where $\mathcal{H}_a = \mathcal{H}_b \equiv \mathcal{H}$, because we do not know if the particle "a" or "b" was destroyed.

We point out that $\dim((\mathcal{H} \otimes \mathcal{H}^0) \oplus (\mathcal{H}^0 \otimes \mathcal{H})) = 2\dim(\mathcal{H} \otimes \mathcal{H}^0)$, so for identical particles we must choose an irreducible subspace of $(\mathcal{H} \otimes \mathcal{H}^0) \oplus (\mathcal{H}^0 \otimes \mathcal{H})$ that corresponds to the space of one-particle states.

3. SUPERTRACES

The partial traces $\mathrm{Tr}_a : \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b) \to \mathrm{End}(\mathcal{H}_b)$ and $\mathrm{Tr}_b : \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b) \to \mathrm{End}(\mathcal{H}_a)$ are widely used in various contexts (see, e.g., Ballentine, 1998; Peres, 1995), but they cannot be used for the description of the destruction. Thus, our purpose is to introduce maps that preserve the trace and map $\mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b)$ to $\mathrm{End}(\mathcal{H}^0 \otimes \mathcal{H}_0)$, $\mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}^0)$ or $\mathrm{End}(\mathcal{H}^0 \otimes \mathcal{H}_b)$.

We can define the following linear map (Caban *et al.*, 2002).

Definition 1. The tensor product *supertrace* $\widehat{\mathrm{Tr}} : \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b) \to \mathrm{End}(\mathcal{H}^0 \otimes \mathcal{H}^0)$ is a linear map such that

$$\widehat{\mathrm{Tr}}(|\chi\rangle\langle\phi| \otimes |\psi\rangle\langle\xi|) = \langle\phi|\chi\rangle\langle\xi|\psi\rangle(|\mathrm{vac}\rangle\langle\mathrm{vac}| \otimes |\mathrm{vac}\rangle\langle\mathrm{vac}|) \tag{4}$$

for any $|\chi\rangle, |\phi\rangle \in \mathcal{H}_a$, and $|\psi\rangle, |\xi\rangle \in \mathcal{H}_b$. Because of linearity, this map is defined on the whole space $\mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b)$.

Next, we need maps that transform the two-particle state into one-particle state. They are given by the following definition (Caban *et al.*, 2002).

Definition 2. The linear maps:

$$\widehat{\mathrm{Tr}}_{\mathrm{L}} : \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b) \to \mathrm{End}(\mathcal{H}^0 \otimes \mathcal{H}_b),$$

$$\widehat{\mathrm{Tr}}_{\mathrm{R}} : \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b) \to \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}^0),$$

$$\widehat{\mathrm{Tr}}_{\mathrm{I}} : \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b) \to \mathrm{End}((\mathcal{H}_a \otimes \mathcal{H}^0) \oplus (\mathcal{H}^0 \otimes \mathcal{H}_b)),$$

$$\widehat{\mathrm{Tr}}_{\mathrm{E}} : \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b) \to \mathrm{End}((\mathcal{H}_a \otimes \mathcal{H}^0) \oplus (\mathcal{H}^0 \otimes \mathcal{H}_b)),$$

called the *left, right, inner, and external partial supertrace*, respectively, act on the endomorphisms of the form $|\chi\rangle\langle\phi| \otimes |\rangle\psi\langle\xi| \in \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b)$ in the following way

$$\widehat{\mathrm{Tr}}_{\mathrm{L}}(|\psi\rangle\langle\chi| \otimes |\phi\rangle\langle\xi|) = \langle\chi|\psi\rangle(|\mathrm{vac}\rangle\langle\mathrm{vac}| \otimes |\phi\rangle\langle\xi|), \tag{5a}$$

$$\widehat{\mathrm{Tr}}_{\mathrm{R}}(|\psi\rangle\langle\chi| \otimes |\phi\rangle\langle\xi|) = \langle\xi|\phi\rangle(|\psi\rangle\langle\chi| \otimes |\mathrm{vac}\rangle\langle\mathrm{vac}|), \tag{5b}$$

$$\widehat{\mathrm{Tr}}_{\mathrm{I}}(|\psi\rangle\langle\chi| \otimes |\phi\rangle\langle\xi|) = \langle\chi|\phi\rangle(|\psi\rangle\langle\mathrm{vac}| \otimes |\mathrm{vac}\rangle\langle\xi|), \tag{5c}$$

$$\widehat{\mathrm{Tr}}_{\mathrm{E}}(|\psi\rangle\langle\chi| \otimes |\phi\rangle\langle\xi|) = \langle\xi|\psi\rangle(|\mathrm{vac}\rangle\langle\chi| \otimes |\phi\rangle\langle\mathrm{vac}|). \tag{5d}$$

Because these superoperators are linear, their action is defined on the whole space $\mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b)$ since every element of $\mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b)$ can be written as the linear combination of the endomorphisms of the form $|\psi\rangle\langle\chi| \otimes |\phi\rangle\langle\xi|$.

We can see from (5c) and (5d) that the internal and external partial supertraces $\widehat{\mathrm{Tr}}_{\mathrm{I}}$ and $\widehat{\mathrm{Tr}}_{\mathrm{E}}$ are nontrivial only for identical particles, i.e., for symmetric or antisymmetric part of $\mathrm{End}((\mathcal{H} \otimes \mathcal{H}^0) \oplus (\mathcal{H} \otimes \mathcal{H}))$ (notice that in this case $\mathcal{H}_a = \mathcal{H}_b \equiv \mathcal{H}$), because in the other case $\langle\chi|\phi\rangle$ and $\langle\xi|\psi\rangle$ must vanish for any $|\psi\rangle, |\chi\rangle \in \mathcal{H}_a$ and $|\phi\rangle, |\xi\rangle \in \mathcal{H}_b$.

4. DISTINGUISHABLE PARTICLES

Now we consider the destruction in two-particle system of distinguishable particles. The apparatus mentioned in Section 1 destroys the particles if the outcomes of measurements of the observables $\hat{\Lambda}_a$ and $\hat{\Lambda}_b$ lie in the subsets Ω_a and Ω_b of spectra Λ_a of $\hat{\Lambda}_a$ and Λ_b of $\hat{\Lambda}_b$, respectively. Let Π_{Ω_a} be the projector onto the subspace of $\mathcal{H}_a$ associated with Ω_a and Π_{Ω_b} be the projector onto the subspace of $\mathcal{H}_b$ associated with Ω_b. Now we perform a simultaneous measurement of the observables $\Pi_{\Omega_a} \otimes I_b$ and $I_a \otimes \Pi_{\Omega_b}$ (I_a and I_b denote the identity operators in $\mathcal{H}_a$ and $\mathcal{H}_b$, respectively). Thus just after the measurement we have the following possible outcomes:

- the measurement of $\Pi_{\Omega_a} \otimes I_b$ and $I_a \otimes \Pi_{\Omega_b}$ both give 0—there are no particles to destroy and the final state is a two-particle state;
- the measurement of $\Pi_{\Omega_a} \otimes I_b$ gives 0 and the measurement of $I_a \otimes \Pi_{\Omega_b}$ gives 1—the particle "*b*" is to destroy and the final state is a one-particle state of the particle "*a*";
- the measurement of $\Pi_{\Omega_a} \otimes I_b$ gives 1 and the measurement of $I_a \Pi_{\Omega_b}$ gives 0—the particle "*a*" is to destroy and the final state is a one-particle state of the particle "*b*";
- the measurement of $\Pi_{\Omega_a} \otimes I_b$ and $I_a \Pi_{\Omega_b}$ both give 1—the particles "*a*" and "*b*" are to destroy and the final state is the vacuum state.

One can easily verify the operators $\Pi^{\perp}_{\Omega_a} \otimes \Pi^{\perp}_{\Omega_b}$, $\Pi^{\perp}_{\Omega_a} \otimes \Pi_{\Omega_b}$, $\Pi_{\Omega_a} \otimes \Pi^{\perp}_{\Omega_b}$, and $\Pi_{\Omega_a} \otimes \Pi_{\Omega_b}$, where $\Pi^{\perp}_{\Omega_a} \equiv I_a - \Pi_{\Omega_a}$ and $\Pi^{\perp}_{\Omega_b} \equiv I_b - \Pi_{\Omega_b}$ are projectors on mutually orthogonal subspaces associated with these cases, appropriately.

Now, to destruct Ω_a- and Ω_b-projected parts of the density matrix ρ we apply appropriately the $\widehat{\mathrm{Tr}}_{\mathrm{L}}(\widehat{\mathrm{Tr}}_{\mathrm{R}})$ to the Ω_a- $(\Omega_b - 1)$ projected part of ρ as well as $\widehat{\mathrm{Tr}}$ to the Ω_a- and Ω_b-projected part, and we arrive at the following definition (Caban *et al.*, 2002).

Definition 3. The *destruction with no selection* in the set Ω of two-particle state $\rho \in \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b)$ of distinguishable particles is defined by the map $D_\Omega : \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b) \to \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}_b) \oplus \mathrm{End}(\mathcal{H}_a \otimes \mathcal{H}^0) \oplus \mathrm{End}(\mathcal{H}^0 \otimes \mathcal{H}_b) \oplus \mathrm{End}(\mathcal{H}^0 \otimes \mathcal{H}^0)$, such that

$$
\begin{aligned}
D_\Omega(\rho) = {} & \left(\Pi^{\perp}_{\Omega_a} \otimes \Pi^{\perp}_{\Omega_b}\right) \rho \left(\Pi^{\perp}_{\Omega_a} \otimes \Pi^{\perp}_{\Omega_b}\right) \\
& + \widehat{\mathrm{Tr}}_{\mathrm{R}} \left[\left(\Pi^{\perp}_{\Omega_a} \otimes \Pi^{\perp}_{\Omega_b}\right) \rho \left(\Pi^{\perp}_{\Omega_a} \otimes \Pi_{\Omega_b}\right)\right] \\
& + \widehat{\mathrm{Tr}}_{\mathrm{L}} \left[\left(\Pi_{\Omega_a} \otimes \Pi^{\perp}_{\Omega_b}\right) \rho \left(\Pi_{\Omega_a} \otimes \Pi^{\perp}_{\Omega_b}\right)\right] \\
& + \widehat{\mathrm{Tr}} \left[\left(\Pi_{\Omega_a} \otimes \Pi_{\Omega_b}\right) \rho \left(\Pi_{\Omega_a} \otimes \Pi_{\Omega_b}\right)\right]. \qquad (6)
\end{aligned}
$$

It can be shown (Caban *et al.*, 2002) that the map D_Ω is a Kraus map.

Now, we illustrate the destruction in two-particle system of distinguishable particles by the following examples.

Example 1. Consider an EPR pair of distinguishable qubits (Galindo and Martín-Delgado, 2002):

$$
|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle \pm |1\rangle \otimes |0\rangle).
$$

Let us assume that $\hat{\Lambda}_a|0\rangle = 0$ and $\hat{\Lambda}_a|1\rangle = |1\rangle$, and similarly $\hat{\Lambda}_b$, so $\Lambda_a = \Lambda_b = \{0, 1\}$. The destruction with no selection takes place if the state of any qubit is $|0\rangle$, so $\Omega_a = \Omega_b = \{0\}$ and, therefore, $\Pi_{\Omega_a} = |0\rangle\langle 0|$ and $\Pi_{\Omega_b} = |0\rangle\langle 0|$. The density matrix for this state is $\rho_{\pm} = |\Psi^{\pm}\rangle\langle\Psi^{\pm}|$ and

$$
D(\rho_{\pm}) = \frac{1}{2}(|1\rangle\langle 1| \otimes |\mathrm{vac}\rangle\langle \mathrm{vac}| + |\mathrm{vac}\rangle\langle \mathrm{vac}| \otimes |1\rangle\langle 1|).
$$

Example 2. Now, consider another EPR pair of distinguishable qubits (Galindo and Martín-Delgado, 2002):

$$
|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle \pm |1\rangle \otimes |1\rangle).
$$

As in the previous example the destruction with no selection takes place if the state of any qubit is $|0\rangle$. The density matrix for this state is now $\rho_{\pm} = |\Phi^{\pm}\rangle\langle\Phi^{\pm}|$ and

$$D(\rho_{\pm}) = \frac{1}{2}(|1\rangle\langle 1| \otimes |1\rangle\langle 1| + |\mathrm{vac}\rangle\langle\mathrm{vac}| \otimes |\mathrm{vac}\rangle\langle\mathrm{vac}|).$$

5. IDENTICAL PARTICLES

Now we consider the destruction in the system of two identical particles. In this case $\mathcal{H}_a = \mathcal{H}_b \equiv \mathcal{H}$. The system of two identical particles is described by a density matrix of the form (1) together with the symmetry conditions (2a) or (2b). As in the previous case, let Π_Ω be the projector onto the subspace of $\mathcal{H}$ associated with $\Omega \subset \Lambda$. Now we perform a measurement of the symmetrized observable $\Pi_\Omega \otimes I + I \otimes \Pi_\Omega$. The spectral decomposition of this observable is

$$\Pi_\Omega \otimes I + I \otimes \Pi_\Omega = 0 \cdot \Pi_\Omega^\perp \otimes \Pi_\Omega^\perp + 1 \cdot \left(\Pi_\Omega^\perp \otimes \Pi_\Omega + \Pi_\Omega \otimes \Pi_\Omega^\perp\right) + 2 \cdot \Pi_\Omega \otimes \Pi_\Omega \quad (7)$$

($\Pi_\Omega^\perp = I - \Pi_\Omega$, as before), where

$\Pi_\Omega^\perp \otimes \Pi_\Omega^\perp$ corresponds to the situation that there is no particle with an eigenvalue of $\hat{\Lambda}$ belonging to Ω,

$\Pi_\Omega^\perp \otimes \Pi_\Omega + \Pi_\Omega \otimes \Pi_\Omega^\perp$ corresponds to the situation that there is exactly one particle with an eigenvalue of $\hat{\Lambda}$ belonging to Ω,

$\Pi_\Omega \otimes \Pi_\Omega$ corresponds to the situation that there are two particles with an eigenvalue of $\hat{\Lambda}$ belonging to Ω.

In view of (7), just after the measurement, we have only the three possibilities:

- the measurement of $\Pi_\Omega \otimes I + I \otimes \Pi_\Omega$ gives 0—there is no particle to destroy and the final state is a two-particle state,
- the measurement of $\Pi_\Omega \otimes I + I \otimes \Pi_\Omega$ gives 1—there is exactly one particle to destroy and the final state is a one-particle state,
- the measurement of $\Pi_\Omega \otimes I + I \otimes \Pi_\Omega$ gives 2—there are two particle to destroy and the final state is the vacuum state.

In order to destruct the Ω-projected part of the density matrix ρ we apply the same algorithm as in the case of distinguishable particles, but now we cannot omit $\widehat{\mathrm{Tr}}$ and $\widehat{\mathrm{Tr}}_{\mathrm{E}}$ because their action is nontrivial. Therefore, we can formulate the following definition (Caban *et al.*, 2002).

Definition 4. The *destruction with no selection* in the set Ω of twoparticle state $\rho \in \mathrm{End}(\mathcal{H} \otimes \mathcal{H})$ of identical particles is defined by the map D_Ω: $\mathrm{End}(\mathcal{H} \otimes \mathcal{H}) \to \mathrm{End}(\mathcal{H} \otimes \mathcal{H}) \oplus \mathrm{End}((\mathcal{H} \otimes \mathcal{H}^0) \oplus (\mathcal{H}^0 \otimes \mathcal{H})) \oplus \mathrm{End}(\mathcal{H}^0 \otimes \mathcal{H}^0)$, such that

$$
\begin{aligned}
D_\Omega(\rho) = {} & \left(\Pi_\Omega^\perp \otimes \Pi_\Omega^\perp\right) \rho \left(\Pi_\Omega^\perp \otimes \Pi_\Omega^\perp\right) \\
& + \widehat{\mathrm{Tr}}_{\mathrm{R}} \left[\left(\Pi_\Omega^\perp \otimes \Pi_\Omega^\perp\right) \rho \left(\Pi_\Omega^\perp \otimes \Pi_\Omega\right)\right] + \widehat{\mathrm{Tr}}_{\mathrm{L}} \left[\left(\Pi_\Omega \otimes \Pi_\Omega^\perp\right) \rho \left(\Pi_\Omega \otimes \Pi_\Omega^\perp\right)\right] \\
& \pm \widehat{\mathrm{Tr}}_{\mathrm{I}} \left[\left(\Pi_\Omega^\perp \otimes \Pi_\Omega\right) \rho \left(\Pi_\Omega \otimes \Pi_\Omega^\perp\right)\right] \pm \widehat{\mathrm{Tr}}_{\mathrm{E}} \left[\left(\Pi_\Omega \otimes \Pi_\Omega^\perp\right) \rho \left(\Pi_\Omega^\perp \otimes \Pi_\Omega\right)\right] \\
& + \widehat{\mathrm{Tr}} \left[(\Pi_\Omega \otimes \Pi_\Omega) \rho (\Pi_\Omega \otimes \Pi_\Omega)\right], \qquad (8)
\end{aligned}
$$

where the signs $+$ and $-$ correspond to symmetric and antisymmetric cases, respectively.

It can be shown (Caban *et al.*, 2002) that this map is a Kraus map if it acts on density matrices obeying the symmetry conditions (2a) or (2b).

Now, we illustrate the destruction in two-particle system of identical particles by the following examples.

Example 3. Now, consider an EPR pair of identical qubits:

$$|\Psi_\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \otimes |1\rangle \pm |1\rangle \otimes |0\rangle),$$

and as previously, let us assume that $\hat{\Lambda}|0\rangle = 0$ and $\hat{\Lambda}|1\rangle = |1\rangle$ and the destruction with no selection takes place if the state of any qubit is $|0\rangle$, so $\Omega = \{0\}$ and, therefore, $\Pi_\Omega = |0\rangle\langle 0|$. The density matrix $\rho_\pm = |\Psi^\pm\rangle\langle\Psi^\pm|$ is symmetric if we choose the sign "$+$" and antisymmetric if we choose "$-$." After the destruction we get

$$D(\rho^\pm) = \frac{1}{2}(|1\rangle \otimes |\mathrm{vac}\rangle + |\mathrm{vac}\rangle \otimes |1\rangle)(\langle 1| \otimes \langle\mathrm{vac}| + \langle\mathrm{vac}| \otimes \langle 1|).$$

Note that in the case of identical qubits in the state $|\Phi^\pm\rangle$ (now $\rho_\pm$ is symmetric for both "$+$" and "$-$") after destruction we get the same state as in the Example 2.

6. CONCLUSIONS

We have given a mathematical formalism that allows one to describe the destruction of a particle from the two-particle state in the framework of quantum mechanics. This is done by means of the reduction procedure (Isham, 1995; Lüders, 1951; von Neumann, 1932) associated with immediate mapping of the part of the reduced density matrix onto vacuum density matrix and is based on the use of supertraces. We point out that the destruction procedure can be treated as a supplement to the von Neumann–Lüders measurement procedure.

The formalism introduced herein should be helpful in a description of the processes when one has the system under time evolution after the destruction. This may happen in the EPR type experiments (the destruction can take place in a detector). For this reason the destruction procedure may also be helpful in quantum information theory. Applications of the destruction procedure to calculation of the EPR quantum correlations will be done in the forthcoming papers.

ACKNOWLEDGMENT

This work is supported by Łódź University Grant No. 267.

REFERENCES

Ballentine, L. E. (1998) *Quantum Mechanics: A Modern Development*, World Scientific, Singapore.

Bohm, D. (1951). *Quantum Theory*, Prentice-Hall, Englewood Clifs, NJ.

Caban, P., Rembieliński, J., Smoliński, K. A., and Walczak, Z. (2002). Destruction of states in quantum mechanics. *Journal of Physics A: Mathematical and General* **35**(14), 3265–3278.

Einstein, A., Podolsky, B., and Rosen, N. (1935). Can quantum-mechanical description of physical reality be considered complete? *Physical Review* **47**(10), 777–780.

Galindo, A. and Martín-Delgado, M. A. (2002). Information and computation: Classical and quantum aspects. *Reviews of Modern Physics* **74**(3), 347–423.

Isham, C. J. (1995). *Lectures on Quantum Theory. Mathematical and Structural Foundations*, Imperial College Press, London.

Lüders, G. (1951). Über die Zustandsänderung durch den Messprozess. *Annalen der Physik* **8**, 322–328.

Peres, A. (1995). *Fundamental Theories of Physics, Vol. 72: Quantum Theory: Concepts and Methods*, Kluwer, Dordrecht, The Netherlands.

von Neumann, J. (1932). *Mathematische Grundlagen der Quantenmechanik*, Springer, Berlin.